

COMPETITION PRACTICE QUESTIONS

One of the things that makes writing math competitions exciting is the mystery: you don't know what kinds of problems they will contain until you open the contest paper on the big day! This means that the best way to prepare is to try solving lots of different kinds of problems. Here are some problems from our Spirit of Math curriculum; they will give you an idea of what you might see on a competition.

Grade 3 & 4 Questions

a) At Odd Ball Junior High School, all the lockers are numbered consecutively from 13 to 99. How many lockers are at Odd Ball Junior High?

b) Hungry Harry can eat 18 donuts in an hour. This is twice as many as his sister Skinny Minny can eat. Together, how long would it take them to eat 81 donuts?

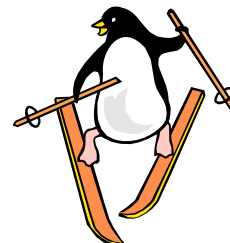


c) There were 8 people at Daylam's birthday party. If they all shook hands with each other, how many handshakes would there be in all?



d) Jimmy loves geometry. His favourite shapes are triangles and pentagons. If he draws 12 figures that are triangles and/or pentagons, using a total of 54 edges, how many of the figures are triangles?

e) There were two dozen penguins skiing down an icy hill. Along came a polar bear and scared all but half a dozen away. How many penguins were not scared away?

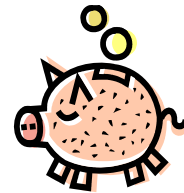


f) How many 2-digit palindromic numbers are there between 10 and 112?

Grade 5 & 6 Questions

a) Peggy scored an average of 68 on her first three tests of the term. What would she have to score on the fourth test to have an average of 70 on the four tests?

b) Joseph has started saving quarters, dimes, nickels and pennies, but is unable to give the exact change for a hamburger that costs \$3.50. What is the greatest amount of money he could have in coins?



c) Joanna prints all of the numbers from 1 to 885. How many times does she print the digit 7?

d) Nonstop trains leave Toronto and Montreal at the same time each day going to the other city along the main line. One train travels at 120 km/h and the other at 90 km/h. Assuming that the track is straight, how far apart are they one hour before they meet?



e) What's the date today if July 7 was six days before one day after four days before the day before eight days after the day before yesterday?

f) Find the smaller angle between the hands of a clock at 6:45.



Grade 7 & 8 Questions

- a) Trina's boat travels 20 km/h in still water. A river has a current that flows at a rate of 5 km/h. Trina travels in her boat downstream for 1 km and then turns around and travels back to her starting point. What is her average rate, in km/h, for the entire trip?



- b) Find the 11 110th digit in the decimal expansion of $\frac{1}{41}$.
- c) If the product $4\,795\,795\,472 \times k$ represents a perfect fourth power, then what is the smallest possible natural value for k ?
- d) A circle has 1500 chords drawn in it so that the maximum number of non-overlapping regions is obtained. Find the difference between the number of points of intersection of the chords and the number of polygons formed by the chords inside the circle.
- e) One country has license plates for cars that have 3 letters followed by 3 digits, or 3 digits followed by 3 letters. A second country has license plates with four letters followed by three digits. Assuming that zero can be the first digit in a three-digit license number, how many more license plates can be made using the 4-letter, 3-digit format than can be made with the 3 and 3 format?
- f) You're standing in a line-up at the movie theatre. If 40% of the people in line are in front of you and $58\frac{1}{3}\%$ of the people in the line are behind you, then what is the total number of people in the queue?





Solutions

Grades 3 & 4

- a) **87** Since the lockers are numbered consecutively from 13 to 99, it is as if the first 12 lockers are 'missing'. Subtract the 'missing' lockers: $99 - 12 = 87$ and there are 87 lockers at the school.
- b) **3 hours** Skinny Minny can eat 9 donuts an hour. Together, she and Harry eat $9 + 18 = 27$ donuts in an hour. It will take $81 \div 27 = 3$ hours for them to eat 81 donuts.
- c) **28** If each person shook every other person's hand then each person would have shaken 7 hands. But this would count each handshake twice (e.g., Daylam shaking Jamie's hand and Jamie shaking Daylam's hand), so the actual number of handshakes in all is $8 \times 7 \div 2 = 28$.
- d) **3** If all 12 figures were triangles, there would be $3 \times 12 = 36$ edges in total. Since we are told there are 54 edges, the extra $54 - 36 = 18$ edges must come from pentagons. Each pentagon has 2 extra edges compared to a triangle, so the 18 extra edges must come from $18 \div 2 = 9$ pentagons. Therefore, there are $12 - 9 = 3$ triangles.
- e) **6** The second sentence can be re-written, "Along came a polar bear and scared all away except half a dozen." Since a dozen is 12, half a dozen is 6. Therefore six penguins were not scared away.
- f) **9** A palindromic number is one that is the same whether you read the digits from right to left or from left to right. The 2-digit palindromic numbers between 10 and 112 are: 11, 22, 33, 44, 55, 66, 77, 88, and 99.



Solutions

Grade 5 & 6

- a) **76** If the average of Peggy's first three test scores was 68, she earned a total of $3 \times 68 = 204$ marks after three tests. To have an average of 70 on her first four tests, she must have a total of $4 \times 70 = 280$ marks after four tests. Therefore, Peggy needs to score $280 - 204 = 76$ on the fourth test.
- b) **\$3.69** Joseph could have as many as thirteen quarters (\$3.25). He could also have as many as four dimes (\$0.40) but not five, as this would make 50 cents which would make a total of \$3.50. He could also have as many as four pennies (\$0.04) but not five. $\$3.25 + \$0.40 + \$0.04 = \3.69 .
- c) **278 times** The digit 7 occurs 20 times within each grouping of 100 numbers (10 times in the ones place and 10 times in the tens place). In the range from 700 to 799 the digit 7 occurs 120 times because there are 100 sevens in the hundreds place. In the range from 800 to 885 there are only 18 more sevens. So, the total number of times that Joanna prints the digit 7 is $20 + 20 + 20 + 20 + 20 + 20 + 120 + 20 + 18 = 278$ times.
- d) **210 km** Working backwards from the meeting point, since one train travels 90 km in one hour and the other 120 km, the trains must be $90 + 120 = 210$ km apart an hour before meeting.
- e) **July 11th** Work backwards. Start with yesterday (Y) and subtract the days before and add the days after (this may also be done on a number line, with yesterday as the "anchor" date). The equation is $Y - 1 + 8 - 1 - 4 + 1 - 6 = 7$. Simplifying gives the equation $Y - 3 = 7$, which means that $Y = 10$. Therefore, yesterday was July 10th and today is July 11th.
- f) **67.5°** The minute hand travels 6° each minute, and the hour hand travels $\frac{1}{2}^\circ$ each minute. At 45 minutes past 6:00, the minute hand has traveled $6 \times 45 = 270^\circ$. At 6:00, the hour hand was at 180° and has since traveled $\frac{1}{2} \times 45 = 22\frac{1}{2}^\circ$, for a total of $202\frac{1}{2}^\circ$. The angle between the hands at 6:45 is therefore $270^\circ - 202\frac{1}{2}^\circ = 67\frac{1}{2}^\circ$



Solutions

Grade 7 & 8

- a) **18.75 km/h** Downstream, Trina travels at a net speed of 25 km/h, so to travel 1 km takes $\frac{1}{25}$ h. Upstream, she travels at a net speed of 15 km/h, taking $\frac{1}{15}$ h to travel 1 km. Her average speed for the 2 km she traveled in total is $2 \div \left(\frac{1}{25} + \frac{1}{15}\right) = 18.75$ km/h.
- b) **9** Since $\frac{1}{41} = 0.\overline{02439}$, the repeating block is five digits long. If we are looking for the 11 110th digit, dividing shows that this is equivalent to $11\ 110 \div 5 = 2\ 222$ blocks of five digits (with no remainder). Since 9 is the 5th digit in the repeating block, it will also be the 11 110th digit.
- c) **1 037 153** The prime factorization of the given number is $2^4 \cdot 13^2 \cdot 17^3 \cdot 19^2$. To make a perfect fourth power, each exponent in the prime factorization must be divisible by four. Therefore, the smallest value for k is $13^2 \cdot 17 \cdot 19^2 = 1\ 037\ 153$.
- d) **1499** Draw several chords and record the results in a chart:

Number of Chords	Points of Intersection	Number of Polygons
0	0	0
1	0	0
2	1	0
3	3	1
4	6	3
5	10	6

Notice that for any number of chords n , the number of points of intersection may be found by adding $1 + 2 + 3 + \dots + (n - 1)$ and the number of polygons is given by $1 + 2 + 3 + \dots + (n - 2)$. For $n = 1500$ chords, the difference between the number of points of intersection and the number of polygons is found from $(1 + 2 + 3 + \dots + 1499) - (1 + 2 + 3 + \dots + 1498) = 1499$.

- e) **421 824 000** Using the 3 and 3 format the number of possible plates is: $26 \times 26 \times 26 \times 10 \times 10 \times 10 \times 2 = 35\ 152\ 000$. Using the 4 by 3 format the number of possible plates is $26 \times 26 \times 26 \times 26 \times 10 \times 10 \times 10 = 456\ 976\ 000$. The difference is 421 824 000. (Notice that the difference is equal to 12 times the number of 3 by 3 plates.)
- f) **60** In front are $\frac{2}{5}$ of the people, and $\frac{7}{12}$ are behind. You represent $1 - \frac{2}{5} - \frac{7}{12} = \frac{1}{60}$ of the people. So there are 60 people in the queue.